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Oscillations in Economic Growth and Renewable Resources with Exogenous Shocks

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Abstract

This paper demonstrates business cycles in a dynamic economic model with physical CapitaLand renewable resources. The model is a synthesis of the neoclassical growth theory and the traditional dynamic models of renewable resources with household behaviour described by an alternative utility function. The model is for a perfectly competitive economy with endogenous wealth physical accumulation, resource change, and division of labour. We simulate the model to demonstrate oscillations with different exogenous shocks. First, we identify the existence of equilibrium points and motion of the dynamic system. Our comparative dynamic analysis shows how the economic system reacts to different periodic changes in the population, the preference and technology.

Keywords: renewable resource, oscillations, wealth accumulation, economic growth,

1. Introduction

The purpose of this study is to develop a dynamic interdependence between capital and resource in a perfectly competitive economy. As far as physical capital and wealth accumulation are concerned, the model in this study is based on the neoclassical growth theory. Most of the models in the neoclassical growth theory are extensions and generalizations of the pioneering works of Solow in 1956. The model has played an important role in the development of economic growth theory by using the neoclassical production function and neoclassical production theory. This study models behaviour of households with an alternative approach proposed by Zhang in the early 1990s (Zhang, 1993). The approach overcomes the lacking of micro foundation for household behaviour in the Solow model and avoids the problems in the Ramsey approach (Zhang, 2005). This study is concerned with dynamic relations of renewable resources with capital accumulation. Stock of renewable resources is changeable according how fast agents utilize resources and how fast renewable resources grow. We integrate the Solow one-sector growth and some neoclassical growth models with renewable resource models (e.g., Plourde, 1971; Stiglitz, 1974; Clark, 1976; Alvarez-Cuadrado and van Long, 2011). As pointed out by Munro and Scott (1985), in the 1950s it was quite difficult to develop workable dynamic models of resources. Solow (1999) argues for the necessity of taking account of natural resources in the neoclassical growth theory. Nevertheless, Solow does not show how to incorporate possible consumption of renewable resource into the growth model. There are only a few models of growth and renewable resources which treat the renewable resource as a source of utility (see, Beltratti, *et al.*, 1994, Ayong Le Kama, 2001). It should be noted that almost all aspects of the model is based on Zhang (2011). A main extension is to introduce time-dependent changes in the preference, the population, and productivities while in the previous model these variables are fixed. The paper is organized as follows. Section 2 introduces the basic model with wealth accumulation and renewable resource dynamics. Section 3 examines dynamic properties of the model and simulates the model, identifying the existence of a unique equilibrium and checking the stability conditions. Section 4 studies effects of changes in some time-dependent parameters on the system. Section 5concludes the study. The appendix proves the analytical results in Section 3.

2 The basic model

The economy has one production sector and one resource sector. Most aspects of the production sector are similar to the standard one-sector growth model. There are only one (durable) good and one renewable resource in the economy under consideration. Households own capital of the economy and distribute their incomes to consume the commodity and renewable resource and to save. Exchanges take place in perfectly competitive markets. We assume a homogenous population. The labor force is distributed between the two sectors. We select commodity to serve as numeraire (whose price is normalized to 1), with all the other prices being measured relative to its price.

The production sector

We use $N_i(t)$ and $K_i(t)$ to stand for the labor force and physical capital of the production sector. The production function is specified as follows

$$F_i(t) = A_i(t) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad A_i, \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \tag{1}$$

where $F_i(t)$ is the output level of the production sector at time t and $A_i(t)$, α_i and β_i are parameters. Markets are competitive; thus labour and capital earn their marginal products. The rate of interest, $r(t)$, and wage rate, $w(t)$, are determined by markets. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \tag{2}$$

where δ_k is the given depreciation rate of physical capital, $0 \leq \delta_k < 1$.

Change of renewable resources

Let $X(t)$ stand for the stock of the renewable resource. The natural growth rate of the resource follows the logistic function of the existing stock

$$\phi_0(t) X(t)\left(1 - \frac{X(t)}{\phi(t)}\right), \quad \phi_0(t), \phi(t) > 0,$$

where the variable, $\phi(t)$, is the maximum possible size for the resource stock, called the carrying capacity of the resource, and the variable, $\phi_0(t)$, is "uncongested" or "intrinsic" growth rate of the renewable resource (e.g., Brander and Taylor, 1998; Cairns and Tian, 2010). Let $F_x(t)$ stand for the harvest rate of the resource. The change rate in the stock is then equal to the natural growth rate minus the harvest rate, that is

$$\dot{X}(t) = \phi_0(t) X(t)\left(1 - \frac{X(t)}{\phi(t)}\right) - F_x(t). \tag{3}$$

We now examine functional form of the harvest rate. We assume a nationally owned open-access renewable resource (Gordon, 1954). Let $N_x(t)$ and $K_x(t)$ stand for, respectively, labour force and capital stocks employed by the resource sector. The harvesting production function is

$$F_x(t) = A_x(t) X^b(t) K_x^{\alpha_x}(t) N_x^{\beta_x}(t), \quad A_x, b \ge 0, \quad \alpha_x, \beta_x > 0, \quad \alpha_x + \beta_x = 1, \quad (4)$$

where $A_x(t)$, b, α_x and β_x are parameters. The specified form implies that if the capital (like machine) and labour inputs are simultaneously doubled, then harvest is also doubled for a given stock of the resource at a given time. This is a generalized form of the Schaefer harvesting production function (Schaefer, 1957). Let $p(t)$ stand for the price of the stock. The marginal conditions are given as follows

$$r(t) + \delta_k = \frac{\alpha_x p(t) F_x(t)}{K_x(t)}, \quad w(t) = \frac{\beta_x p(t) F_x(t)}{N_x(t)}. \quad (5)$$

Let $N(t)$ and $K(t)$ stand for respectively the (exogenous) population and total capital stock. The labour force is allocated between the two sectors. As full employment of labour and capital is assumed, we have

$$K_i(t) + K_x(t) = K(t), \; N_i(t) + N_x(t) = N(t). \quad (6)$$

Consumer's behaviour

We denote per capita wealth by $k(t)$, where $k(t) \equiv K(t)/N(t)$. Per capita current income from the interest payment $r(t)k(t)$ and the wage payment $w(t)$ is given by

$$y(t) = r(t)k(t) + w(t).$$

We call $y(t)$ the current income. The per capita disposable income is given by

$$\hat{y}(t) = y(t) + k(t) = (1 + r(t))k(t) + w(t). \quad (7)$$

A consumer would distribute the total available budget among saving, $s(t)$, consumption of the commodity, $c(t)$, and consumption of the resource good, $c_x(t)$. The budget constraint is

$$c(t) + s(t) + p(t)c_x(t) = \hat{y}(t). \quad (8)$$

We assume that consumers' utility function is a function of $s(t)$, $c(t)$, and $c_x(t)$, as follows

$$U(t) = c^{\xi_0(t)}(t) s^{\lambda_0(t)}(t) c_x^{\chi_0(t)}(t), \quad \xi_0(t), \lambda_0(t), \chi_0(t) > 0, \quad (9)$$

where $\xi_0(t)$ is called the propensity to consume, $\lambda_0(t)$ the propensity to own wealth, and $\chi_0(t)$ the propensity to consume the resource good. These variables are exogenous. Maximizing $U(t)$ in (9) subject to budget constraint (8) yields

$$c(t) = \xi(t)\hat{y}(t), \quad s(t) = \lambda(t)\hat{y}(t), \quad p(t)c_x(t) = \chi(t)\hat{y}(t), \tag{10}$$

where

$$\xi(t) \equiv \rho(t)\xi_0(t), \quad \lambda(t) \equiv \rho(t)\lambda_0(t), \quad \chi(t) \equiv \rho(t)\chi_0(t),$$
$$\rho(t) \equiv \frac{1}{\xi_0(t) + \lambda_0(t) + \chi_0(t)}.$$

Wealth accumulation

According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{k}(t) = s(t) - k(t). \tag{11}$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

Market equilibrium

The demand for and supply of the resource balance at any point of time

$$c_x(t)N(t) = F_x(t). \tag{12}$$

As output of the production sector is equal to the sum of the level of consumption, the depreciation of capital stock and the net savings, we have

$$C(t) + S(t) - K(t) + \delta_k K(t) = F_i(t), \tag{13}$$

where $S(t) - K(t) + \delta_k K(t)$ is the sum of the net saving and depreciation and

$$C(t) = c(t)N(t), \quad S(t) = s(t)N(t).$$

We thus built the model.

3 The dynamics and its properties

This section examines dynamics of the model. First, we introduce a new variable by $z(t) \equiv K_i(t)/K_x(t)$.

Lemma

The economy is governed by the following two differential equations

$$\dot{z}(t) = \left(\lambda(t)N(t)\hat{y}(t) - K(t) - k(t)\dot{N}(t) - \frac{\partial \Lambda}{\partial t}\right)\left(\frac{\partial \Lambda}{\partial z}\right)^{-1},$$

$$\dot{X}(t) = \phi_0(t)X(t)\left(1 - \frac{X(t)}{\phi(t)}\right) - F_x(z(t), X(t), t), \tag{14}$$

where the functions in (14) are functions of t, $z(t)$ and $X(t)$ explained in the appendix. Moreover, all the other variables can be determined as functions of t, $z(t)$ and $X(t)$ at any point in time by the following procedure: $K(t)$ by (A9) $\rightarrow$ $K_i(t)$ and $K_x(t)$ by (A2) $\rightarrow$ $N_i(t)$ and $N_x(t)$ by (A3) $\rightarrow$ $F_i(t)$ by (1) $\rightarrow$ $r(t)$ and $w(t)$ by (2) $\rightarrow$ $F_x(t)$ by (4) $\rightarrow$ $p(t)$ by (5) $\rightarrow$ $\hat{y}(t)$ by (7) $\rightarrow$ $c(t)$, $c_x(t)$ and $s(t)$ by (10).

As the expressions of the analytical results are tedious, for illustration we specify the parameter values and simulate the model. First we are concerned with the case that all the parameters are time-independent. We specify the parameters as follows

$$N_0 = 5, \ \ \alpha_i = 0.3, \ \ A_i = 1, \ \ \alpha_x = 0.4, \ \ A_x = 0.5, \ \ \phi = 1, \ \ \phi_0 = 6,$$
$$\lambda_0 = 0.6, \ \ \xi_0 = 0.15, \ \ \chi_0 = 0.06, \ \ \delta_k = 0.05. \tag{15}$$

The capacity is unity and the adjustment speed, ϕ_0 , is fixed at 6. The population is fixed at 5. The propensity to save is much higher than the propensity to consume the commodity and the propensity to consume the renewable resource. The equilibrium values are given as in (16).

$$K = 18.55, \ \ X = 0.82, \ \ F_i = 5.57, \ \ F_x = 0.88, \ \ N_i = 3.89, \ \ N_x = 1.11,$$
$$K_i = 12.84, \ \ K_x = 5.71, \ \ p = 2.11, \ \ r = 0.08, \ \ w = 1.00, \ \ c_x = 0.18,$$
$$c = 0.83. \tag{16}$$

The two eigenvalues are -4.93 and -0.20. This guarantees the stability of the steady state. Hence, the dynamic system has a unique stable steady state. With the initial conditions, $z(0) = 1.5$ and $X(0) = 0.5$, we plot the motion of the system as in Figure 1. We see that the level of the resource stocks is increased and its price falls over time. The capital and labor force employed by the resource sector fall over time.

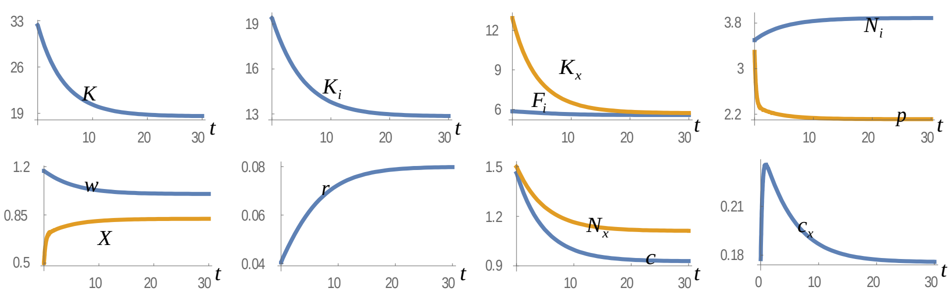

Figure 1: Motion of the Economic System

4. Comparative dynamic analysis

This section examines effects of changes in some parameters on the motion of the economic system.

Periodic changes in the propensity to consume the resource

First, we study the case the propensity to consume the resource is changed in the following way

$$\chi_0(t) = 0.06 + 0.01\sin(t).$$

The simulation results are demonstrated in Figure 2. In the plots, a dashed line stands for the motion of a variable in Figure 1, while a solid line represents the motion of change of the variable under the periodic shocks. If we consider the parameters (15) and variable values in Figure 1 as the long-term trend values, we see that the perturbations cause the system to oscillate around the trends. We observe that when the propensity to consume the resource is increased, the national and production sector's capital stocks, wage rate and level of the consumption good tend to fall, the rate of interest tends to rise.

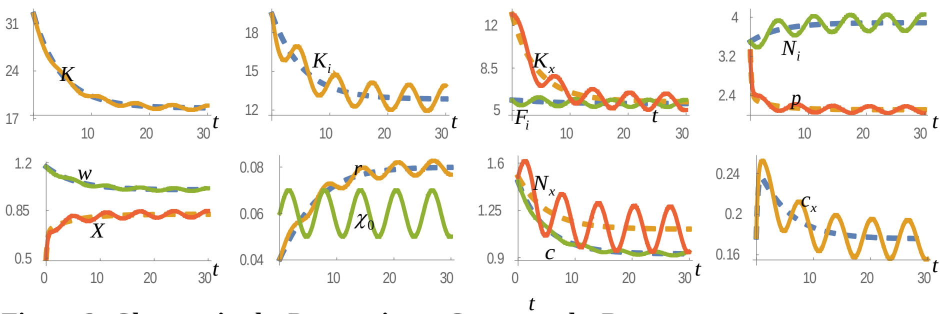

Figure 2: Changes in the Propensity to Consume the Resource

Changes in the propensity to consume goods

We now allow the propensity to consume goods to change as follows

$$\xi_0(t) = 0.15 + 0.04\sin(t).$$

The simulation results are demonstrated in Figure 3.

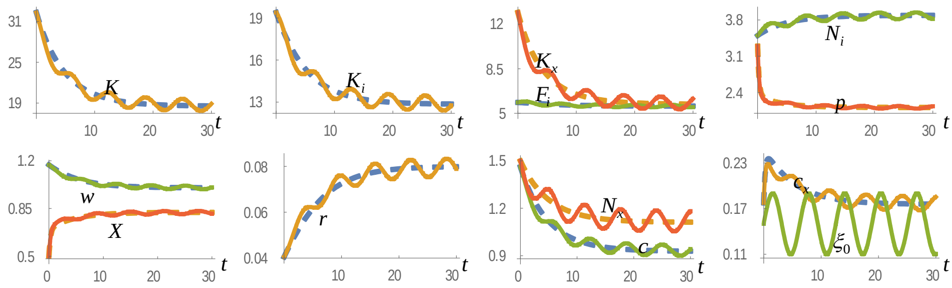

Figure 3: Changes in the Propensity to Consume Goods

Changes in the propensity to consume goods

We now allow the population to change as follows

$$N(t) = 5 + 0.2\sin(t).$$

The simulation results are demonstrated in Figure 4.

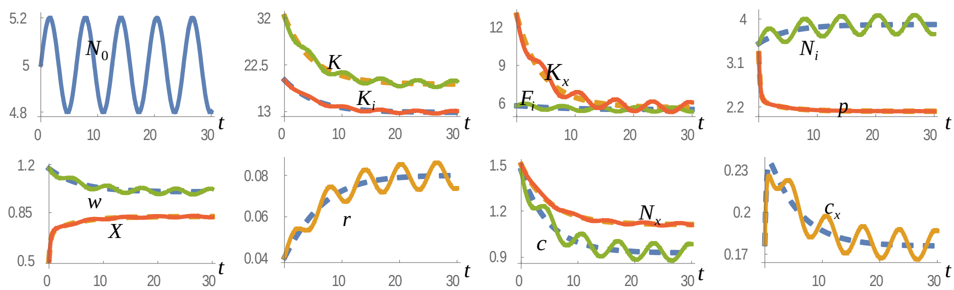

Figure 4: The Impact of Population Change

5. Concluding Remarks

This paper is to further examine dynamic behaviour of the dynamic economic model with physical capital and renewable resources proposed by Zhang (2011). Different from Zhang's study, this study shows how the dynamic system reacts to different time dependent exogenous shocks. The model is a synthesis of the neoclassical growth theory and the traditional dynamic models of renewable resources with household behaviour described by an alternative utility function. It is for a perfectly competitive economy with endogenous wealth physical accumulation, resource change, and division of labour. We simulated the model to demonstrate oscillations with different exogenous shocks. First, we identified the existence of equilibrium points and motion of the dynamic system. Our comparative dynamic analysis showed how the economic system reacts to different periodic changes in the population, the preference and technology.

Appendix: Proving Lemma 1

From (2) and (5), we obtain

$$z = \frac{K_i}{K_x} = \alpha \frac{N_i}{N_x}, \text{(A1)}$$

where we omit time index and $\alpha \equiv \beta_x \alpha_i / \alpha_x \beta_i$. By (A1) and (6), we solve

$$K_i = \frac{zK}{z+1}, \quad K_x = \frac{K}{z+1}, \text{(A2)}$$

$$N_i = \frac{zN}{z+\alpha}, \quad N_x = \frac{\alpha N}{z+\alpha}. \text{(A3)}$$

By (12) and $p c_x = \chi \hat{y}$ in (10), we have

$$\chi N \hat{y} = p F_x. \text{ (A4)}$$

By the definition of $\hat{y}$, we have

$$N \hat{y} = \left(\delta + \frac{\alpha_x p F_x}{K_x} \right) K + \frac{\beta_x p F_x N}{N_x}, \qquad \text{(A5)}$$

where we use (5) and $\delta \equiv 1 - \delta_k$. Insert (A5) in (A4)

$$\left[\frac{1}{\chi} - (1 + z)\alpha_x - \frac{(z + \alpha)\beta_x}{\alpha}\right] p F_x = \delta K, \text{(A6)}$$

where we also use (A2) and (A3). From (2) and (5), we solve

$$p F_x = \frac{\beta_i N_x F_i}{\beta_x N_i}. \text{(A7)}$$

Substituting (A7) into (A6) yields

$$\left[\frac{1}{\chi} - (1 + z)\alpha_x - \frac{(z + \alpha)\beta_x}{\alpha}\right] \frac{\alpha \beta_i F_i}{\beta_x z} = \delta K, \text{(A8)}$$

where we use (A1). Substituting (1), (A3) and (A2) into (A8), we solve

$$K = \Lambda(z, t) \equiv \left(\frac{1}{z + 1}\right)^{\alpha_i / \beta_i} \frac{n_0 (n_1 - n_2 z)^{1/\beta_i}}{z + \alpha}, \text{(A9)}$$

where

$$n_0 \equiv N\left(\frac{\beta_i A_i}{\beta_x \delta}\right)^{1/\beta_i} > 0, \;\; n_1 \equiv \frac{\alpha}{\chi} - \alpha > 0, \;\; n_2 \equiv \alpha \alpha_x + \beta_x = \frac{\beta_x}{\beta_i} > 0.$$

We express K as a function of t and z. From (A2), K_i and K_x are functions of t and z. From (A3), N_i and N_x are functions of t and z. By the procedure we can express other variables as functions of t, $z(t)$ and $X(t)$ at any point in time. Taking derivatives of (A9) with respect to t yields

$$\dot{K} = \frac{\partial \Lambda}{\partial z}\dot{z} + \frac{\partial \Lambda}{\partial t}, \text{(A10)}$$

where

$$\frac{\partial \Lambda}{\partial z} = -\left[\frac{\alpha_i}{\beta_i(z+1)} + \frac{1}{z+\alpha} + \frac{n_2}{\beta_i(n_1 - n_2 z)}\right]\Lambda < 0.$$

Multiplying the two sides of (11) with N and using (10), we have

$$\dot{K} = \lambda N \hat{y} - K - k\dot{N}. \text{(A11)}$$

Substitute (A10) and $K = \Lambda(z)$ into the above function,

$$\dot{z} = \left(\lambda N \hat{y} - K - k\dot{N} - \frac{\partial \Lambda}{\partial t}\right)\left(\frac{\partial \Lambda}{\partial z}\right)^{-1}. \qquad \text{(A12)}$$

We prove the lemma.

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